MATH 254 for MECH 222 is an introduction to multivariable and vector calculus with applications in mechanical engineering. The course is divided into three main topics: multiple integrals, partial derivatives and vector calculus. We explore applications in thermodynamics, fluid dynamics and numerical methods using MATLAB.

## **Textbooks**

- APEX: APEX Calculus, by Gregory Hartman (Version 4.0)
- GUI: Multivariable Calculus: Early Transcendentals, by David Guichard et al.

## Lecture Schedule & Exercises

#	Topics	Exercises
1	· Double integrals over rectangles	APEX §13.1: 5, 6
	· Fubini's theorem and iterated integrals	GUI §15.1: 1, 2
2	· Double integrals over general regions	APEX §13.1: 7, 9, 11, 13, 17, 19
	· Switching the order of integration	APEX §13.2: 7, 9, 13, 17, 19, 21, 25
		GUI §15.1: 3, 6, 9, 12, 15, 18, 21, 24, 32
3	· Double integrals in polar coordinates	APEX §13.3: 3, 5, 7, 9, 13, 15
		GUI §15.2: 3, 6, 9, 12, 15, 18
4	· Centre of mass in 2D	APEX §13.4: 11, 15, 19, 21, 23, 27, 29
	· Moment of inertia in 2D	GUI §15.3: 1, 4, 7, 10, 13
5	· Mass flow rate	See notes on Canvas
	· Hydrostatic pressure	
6	· Triple integrals	APEX §13.6: 5, 7, 9, 11, 15, 17, 19, 21, 23
	· Center of mass in 3D	GUI §15.5: 3, 5, 8, 10, 12, 14, 16
7	· Triple integrals in cylindrical coordinates	APEX §13.7: 11, 13, 15, 23, 25, 27, 29
	· Moment of inertia in 3D	
8	· Triple integrals in spherical coordinates	APEX §13.7: 5, 7, 9, 17, 19, 31, 33, 35, 37
	· Center of buoyancy	GUI §15.6: 3, 5, 7, 9, 11, 13, 15
9	· Partial derivatives	APEX §12.3: 5, 7, 9, 13, 17, 21, 25, 29, 33
	· Chain rule	APEX §12.5: 7, 9, 13, 17, 21, 25, 29
	· Material derivative	GUI §14.3: 1, 3, 5, 7
		GUI §14.4: 1, 3, 5, 7, 8
		GUI §14.6: 1, 5, 9, 10, 11
10	· Directional derivatives and gradient	APEX §12.6: 7, 9, 11, 15, 17, 19, 23, 25, 27
11	· Partial differential equations	See notes Canvas
	· Navier-Stokes equations	
	· Heat equation	
12	· Tangent planes and linearization	APEX §12.7: 5, 9, 13, 17, 21, 23
		GUI §14.3: 8, 9, 11, 12
		GUI §14.5: 2, 5, 8, 10, 12, 14, 16, 18, 19, 21

13	· Critical points and optimization	APEX §12.8: 5, 7, 9, 11, 13, 15, 17
		GUI §14.7: 1, 6, 14, 15, 16, 17, 18
14	· Constrained optimization	GUI §14.8: 4, 5, 7, 9, 10, 13, 14, 17
	· Lagrange multipliers	
15	· Mid-semester review	
16	· Parameterizations of surfaces	APEX §14.5: 3, 5, 7, 9, 11, 13, 15
		GUI §16.6: 1, 2
17	· Surface area	APEX §14.5: 17, 19, 21, 23
		GUI §16.6: 3, 5, 7, 9, 12, 15
18	· Surface integrals	APEX §14.6: 5, 6
	· Center of mass of a surface	GUI §16.7: 1, 2, 3, 4
19	· Vector fields	APEX §14.2: 5, 7, 9, 11, 13, 15, 17
	· Divergence and curl	GUI §16.1: 1, 3, 5
20	· Flux integrals	APEX §14.6: 7, 9, 11, 13
	· Momentum flux	GUI §16.7: 6, 8, 10, 11
21	· Divergence theorem	APEX §14.7: 5, 7, 13, 15, 21
		GUI §16.9: 2, 4, 6, 8, 10, 12
22	· Proof of divergence theorem	See notes on Canvas
	· Archimedes principle and buoyancy	
23	· Curves in space	APEX §11.1: 5, 9, 12, 15, 17, 19, 21, 25
	· Arc length	APEX §11.2: 12, 13, 14, 17, 21, 39, 41
		GUI §13.1: 1, 3, 7, 9
		GUI §13.2: 2, 7, 11, 12, 15, 17
		GUI §13.3: 1, 3, 4
24	· Line integrals	APEX §14.3: 7, 9, 11
	· Center of mass of a wire	GUI §16.2: 3, 5, 7, 9, 11, 13, 15, 17, 19, 21
25	· Line integrals of vector fields	APEX §14.3: 27, 19, 21
	$\cdot$ Fundamental theorem of line integrals	GUI §16.3: 2, 4, 6, 8, 10
26	· Stokes theorem	APEX §14.7: 9, 11, 17, 19
		GUI §16.8: 1, 2, 3, 4, 7
27	· Proof of Stokes theorem	See notes on Canvas
28	· Advanced applications	See notes on Canvas