## MATH 254 for MECH 222

MATH 254 for MECH 222 is an introduction to multivariable and vector calculus with applications in mechanical engineering. The course is divided into three main topics: multiple integrals, partial derivatives and vector calculus. We explore applications in thermodynamics, fluid dynamics and numerical methods using MATLAB.

## Textbooks

- APEX: APEX Calculus, by Gregory Hartman (Version 4.0)
- GUI: Multivariable Calculus: Early Transcendentals, by David Guichard et al.


## Lecture Schedule \& Exercises

| \# | Topics | Exercises |
| :---: | :---: | :---: |
| 1 | - Double integrals over rectangles <br> - Fubini's theorem and iterated integrals | $\begin{aligned} & \text { APEX } \S 13.1: 5,6 \\ & \text { GUI } \S 15.1: 1,2 \end{aligned}$ |
| 2 | - Double integrals over general regions <br> - Switching the order of integration | APEX §13.1: 7, 9, 11, 13, 17, 19 <br> APEX §13.2: 7, 9, 13, 17, 19, 21, 25 <br> GUI §15.1: $3,6,9,12,15,18,21,24,32$ |
| 3 | - Double integrals in polar coordinates | APEX §13.3: 3, 5, 7, 9, 13, 15 GUI §15.2: $3,6,9,12,15,18$ |
| 4 | - Centre of mass in 2D <br> - Moment of inertia in 2D | APEX §13.4: 11, 15, 19, 21, 23, 27, 29 GUI §15.3: 1, 4, 7, 10, 13 |
| 5 | - Mass flow rate <br> - Hydrostatic pressure | See notes on Canvas |
| 6 | - Triple integrals <br> - Center of mass in 3D | APEX §13.6: 5, 7, 9, 11, 15, 17, 19, 21, 23 GUI §15.5: $3,5,8,10,12,14,16$ |
| 7 | - Triple integrals in cylindrical coordinates <br> - Moment of inertia in 3D | APEX §13.7: $11,13,15,23,25,27,29$ |
| 8 | - Triple integrals in spherical coordinates <br> - Center of buoyancy | APEX §13.7: $5,7,9,17,19,31,33,35,37$ GUI §15.6: 3, 5, 7, 9, 11, 13, 15 |
| 9 | - Partial derivatives <br> - Chain rule <br> - Material derivative | APEX §12.3: 5, 7, 9, 13, 17, 21, 25, 29, 33 <br> APEX §12.5: 7, 9, 13, 17, 21, 25, 29 <br> GUI §14.3: $1,3,5,7$ <br> GUI §14.4: $1,3,5,7,8$ <br> GUI §14.6: $1,5,9,10,11$ |
| 10 | - Directional derivatives and gradient | APEX §12.6: 7, 9, 11, 15, 17, 19, 23, 25, 27 |
| 11 | - Partial differential equations <br> - Navier-Stokes equations <br> - Heat equation | See notes Canvas |
| 12 | - Tangent planes and linearization | APEX §12.7: 5, 9, 13, 17, 21, 23 <br> GUI §14.3: $8,9,11,12$ <br> GUI $\S 14.5: 2,5,8,10,12,14,16,18,19,21$ |


| 13 | - Critical points and optimization | APEX $\S 12.8: 5,7,9,11,13,15,17$ GUI §14.7: $1,6,14,15,16,17,18$ |
| :---: | :---: | :---: |
| 14 | - Constrained optimization <br> - Lagrange multipliers | GUI §14.8: $4,5,7,9,10,13,14,17$ |
| 15 | - Mid-semester review |  |
| 16 | - Parameterizations of surfaces | APEX §14.5: 3, 5, 7, 9, 11, 13, 15 GUI §16.6: 1,2 |
| 17 | - Surface area | APEX §14.5: 17, 19, 21, 23 GUI §16.6: $3,5,7,9,12,15$ |
| 18 | - Surface integrals <br> - Center of mass of a surface | APEX §14.6: 5, 6 <br> GUI §16.7: $1,2,3,4$ |
| 19 | - Vector fields <br> - Divergence and curl | APEX §14.2: 5, 7, 9, 11, 13, 15, 17 GUI §16.1: 1, 3, 5 |
| 20 | Flux integrals <br> - Momentum flux | APEX §14.6: 7, 9, 11, 13 GUI §16.7: $6,8,10,11$ |
| 21 | - Divergence theorem | APEX §14.7: 5, 7, 13, 15, 21 GUI §16.9: 2, 4, 6, 8, 10, 12 |
| 22 | - Proof of divergence theorem <br> - Archimedes principle and buoyancy | See notes on Canvas |
| 23 | - Curves in space <br> - Arc length | APEX §11.1: 5, 9, 12, 15, 17, 19, 21, 25 <br> APEX §11.2: 12, 13, 14, 17, 21, 39, 41 <br> GUI §13.1: 1, 3, 7, 9 <br> GUI §13.2: $2,7,11,12,15,17$ <br> GUI §13.3: 1, 3, 4 |
| 24 | - Line integrals <br> - Center of mass of a wire | APEX §14.3: 7, 9,11 GUI §16.2: $3,5,7,9,11,13,15,17,19,21$ |
| 25 | - Line integrals of vector fields <br> - Fundamental theorem of line integrals | $\begin{aligned} & \text { APEX §14.3: } 27,19,21 \\ & \text { GUI } \S 16.3: 2,4,6,8,10 \end{aligned}$ |
| 26 | - Stokes theorem | APEX §14.7: 9, 11, 17, 19 GUI §16.8: $1,2,3,4,7$ |
| 27 | - Proof of Stokes theorem | See notes on Canvas |
| 28 | - Advanced applications | See notes on Canvas |

